Grade 7 Probability

7.PR.4, 5, & 6	
Express probabilities as ratios, fractions, and percents.	1. Determine the probability of an outcome occurring for a given probability experiment, and express it as a ratio, fraction, and percent. 2. Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).
Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.	3. Provide an example of two independent events, such as (i) spinning a 4-section spinner and an 8-sided die (ii) tossing a coin and rolling a twelve-sided die (iii) tossing two coins (iv) rolling two dice and explain why they are independent. 4. Identify the sample space (all possible outcomes) for each of two independent events using a tree diagram, table, or another graphic organizer.
Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or another graphic organizer) and experimental probability of two independent events.	5. Determine the theoretical probability of an outcome involving two independent events. 6. Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability to the theoretical probability. 7. Solve a probability problem involving two independent events.

Clarification of the outcome:

- ✦ The three outcomes are combined because they are strongly inter-related.
- ✦ In grade 6, students quantified probability for simple events such as throwing ONE die. In grade 7, two events such as throwing TWO dice or spinning TWO spinners are involved, where the sample space is used to determine the theoretical probability.
- ✦ Independent events are events that have NO influence on each other (e.g. throwing two dice). A sample space refers to ALL possible outcomes of the event(s) (e.g. all ways two dice can land; all positions a spinner can stop at).

Required close-to-at-hand prior knowledge:

- ✤ Understand theoretical and experimental probability, and quantifying probability (grade 6 matters).
- ✤ Comfortable with fractions and percents, and the relationship between these ways of writing a number. Comfortable the connection between a fraction and ratio (e.g. 1/3 can also be seen as the ratio 1: 3).

SET SCENE stage

The problem task to present to students:

Have students visit website, Extrasensory Perception Test / **ESP test** / Intuition test, and take the test at least ten times, recording the number of successes. Have students determine the theoretical and experimental probabilities of getting the symbols correct.

Comments:

The task is a revisit of grade 6 probability concepts - experimental and theoretical probability. An ESP activity is a good way to encourage engagement in these concepts.

- ☐ You could design your own ESP test activity involving one event and use it instead if Internet connection is not possible.

DEVELOP stage

Activity 1: Revisits SET SCENE, and addresses achievement indicators 1 and 4.

- Have students explain what the theoretical and experimental probabilities are for the ESP experiment. Discuss whether they have ESP powers, based on those probabilities.

Activity 2: Addresses achievement indicators 1 and 2.

- Prepare a bag ahead of time that contains 10 objects of the same colour (e.g. all red unifix cubes). Have a student pull an object out of the bag, returning it into the bag. Repeat 20 times in all. Use a tally chart to record the results. Have the students express the experimental probability of pulling out the red object as a ratio, fraction, and percent. The three values should be equivalent to 1 (e.g. 20:20, 20/20, and 100%). Ask students to guess what is in the bag. Let them see the contents of the bag. Ask them whether it was certain that the red object would be pulled out. Discuss the probability for a certain event as being equivalent to 1 (e.g. 20:20, 20/20, 100%). Ask them the theoretical probability of pulling a black object out of the bag. Discuss the probability of an impossible event as being equivalent to 0 (e.g. 0:20, 0/20, 0%).
- Ask students to describe certain-to-happen events in the real world and to express the probability of them occurring as a ratio, fraction, and percent (all values should be equivalent to 1).
- Ask students to describe impossible-to-happen events in the real world and to express the probability of them occurring as a ratio, fraction, and percent (all values should be equivalent to 0).

Activity 3: Addresses achievement indicator 1.

- Prepare a bag ahead of time that contains 10 objects of the varying colours (e.g. 3 red unifix cubes, 2 white, 5 green). Have a student pull an object out of the bag, returning it into the bag. Repeat 20 times in all. Use a tally chart to record the results. Suppose the red object (e.g.) was pulled out 5 times. Have the students express the experimental probability of pulling out the red object as a ratio, fraction, and percent. The three values should be equivalent (e.g. 5:20, 5/20, and 25%). Ask students to guess what is in the bag.
- Reveal the contents of the bag. Ask students to express the theoretical probability of pulling out each colour. For the bag described here, the theoretical probabilities should be: RED - 3:10, 3/10, 30%; WHITE - 2:10, 2/10, 20%; GREEN - 5:10, 5/10, 50%.

Activity 4: Addresses achievement indicator 3.

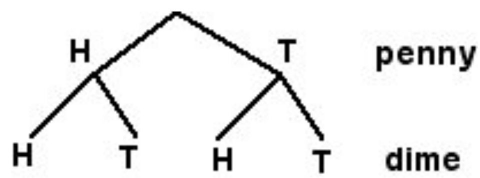
- Ask students to imagine throwing a dime and a penny. Ask them whether getting a HEAD with the dime will affect how the penny lands. Ask them whether getting a HEAD with the penny will affect how the dime lands. Discuss that the two events are **independent events**. They do not influence each other.
- Ask students to imagine throwing a 6-sided die and a quarter. Ask them whether getting a HEAD with the quarter will affect how the die lands. Ask them whether getting a '4' with the die will affect how the quarter lands. Ask if the two events are independent events. ENSURE students realize that they are independent events because they do not influence each other.
- Ask students to imagine throwing two identical dice. Ask them whether getting a '6' with one of the die will affect how the second die lands. Ask them whether getting a '4' with the other die will affect how the first lands. Ask if the two events are independent events. ENSURE students realize that they are independent events.
- Ask students to imagine picking a marble from a bag where there are two marbles in the bag: a red one and a black one. You pick a marble but DO NOT put in back into the bag. Ask them whether picking the red marble first will affect the colour of the second marble picked (yes, it must be black). Ask them whether picking the black marble first will affect the colour of the second marble picked (yes, it must be red). Discuss that the two events are NOT independent events. Ask them how the situation could concern independent events. Lead them to realize that if we put the marble back into the bag after picking it, then the two events are independent events.

Activity 5: Addresses achievement indicators 1, 3, 4, 5, and 7.

- With teacher assistance, have students list in a table all possible ways a penny and a dime can land (see table). Discuss how many events and whether they are independent. [There are 2 independent events - flip PENNY and flip DIME.] ENSURE students realize that H-T and T-H are different possibilities. Introduce the term **sample space**: all possible outcomes of one or more events.

PENNY	DIME
HEAD	HEAD
HEAD	TAIL
TAIL	HEAD
TAIL	TAIL

- Have students make a tree diagram (see diagram) that shows all the ways that two coins can land.



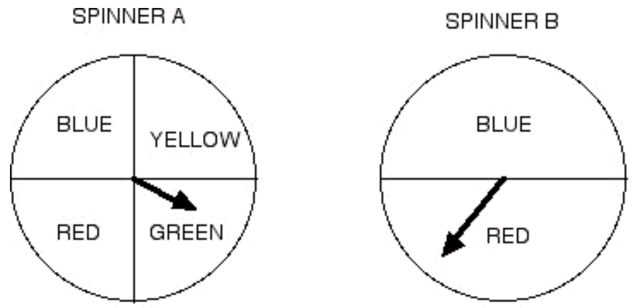
- Have students determine the theoretical probability of: 0 HEADs showing, 1 HEAD showing, and 2 HEADs showing. Ensure that students understand the theoretical probability of zero HEADS showing is 1/4; one HEAD showing is 2/4; and two HEADs showing is 1/4. Discuss the probabilities values as to how they are derived from the sample space.

Note:

A penny and a dime are used intentionally. It helps students realize that penny-TAIL and dime-HEAD landing is different from penny-HEAD and dime-TAIL landing.

Activity 6: Addresses achievement indicators 1, 3, 4, 5, and 7, and practice.

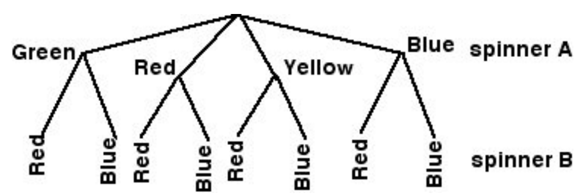
- Provide students with two different spinners (see diagram). Discuss whether spinning the two spinners is an example of two independent events.



- Have students list in a table all possible ways the two spinners can stop. Discuss the term 'sample space' in relation to the table.

SPINNER A	SPINNER B
BLUE	BLUE
BLUE	RED
YELLOW	BLUE
YELLOW	RED
GREEN	BLUE
GREEN	RED
RED	BLUE
RED	RED

- Have students use a tree diagram to show all possible ways the two spinners can stop. Discuss the term 'sample space' in relation to the tree diagram.



- Have students use the sample space to determine the theoretical probability of each outcome. For example, the theoretical probability of both spinners stopping on BLUE is 1/8; the probability of both spinners stopping on YELLOW is 0/8; the probability of both spinners stopping on BLUE/RED is 2/8, and so on. ENSURE students understand how to use the sample space to determine probabilities.

Activity 7: Addresses achievement indicators 1, 4, 5, 6, and 7.

- Have students solve the following problem by determining the sample space (e.g., red-red; red-blue, etc.) by listing in a table or by making a tree diagram.

  A carnival has a game that involves picking a coloured ball (red, blue, or black) from one hat and picking a coloured ball (red, blue, yellow, or black) from a second hat. You are blindfolded to play the game. You win a prize if the two balls you picked are the same colour. What is the probability of winning a prize?

- Ensure that students understand that the theoretical probability of winning is 4/20 and that fraction is equivalent to 20%.
- Organize students into groups. Have them make the two hats and their contents, and then have them play the carnival game 20 times.
- Have students determine the experimental probability as a fraction and as a percent and compare it to the theoretical probability. Discuss the results.

Activity 8: Addresses achievement indicators 1, 4, 5, 6, and 7, and practice.

- Provide two 4-sided dice (having 1 to 4 on them). Discuss the different sums that are possible for the two numbers landing face down when the two dice are thrown (for example, a sum of 6 is possible: 2 + 4, 4 + 2, and 3 + 3 each add up to 6).
- Have students list in a table all possible ways the two 4-sided dice can land face down and determine the theoretical probabilities (as a fraction and as a percent) of a sum of: 2, 3, 4, 5, 6, 7, and 8 occurring. For example a sum of 2 can occur in one way (1 + 1); a sum of 3 can occur in two ways (1 + 2, 2 + 1); a sum of 4 can occur in three ways (1 + 3, 3 + 1, 2 + 2); etc. ENSURE that students understand that the theoretical probability of a sum of 2 occurring is 1/16; the probability of a sum of 3 is 2/16; the probability of a sum of 4 is 3/16; and so on.
- Have students throw the 4-sided dice 20 times, recording the results. Have students determine the experimental probabilities (as a fraction and as a percent) of a sum of 2 occurring, a sum of 3, a sum of 4, and so on. Have them compare the theoretical probabilities to the experimental probabilities.

Note:

The reason for 'face down' is that a 4-sided die is a triangular pyramid (pointy on top). When it lands, the number "showing" is the number that is face down on the table.

Activity 9: Addresses achievement indicators 1, 4, 5, 6, and 7, and practice.

- ✦ Have students list in a table (or by making a tree diagram) all possible ways two 6-sided dice (having 1 to 6 on them) can land face up and determine the theoretical probability of the two numbers showing being the same (a 1 and a 1; a 2 and a 2; etc.). Ensure that students understand that the theoretical probability of both numbers being the same is 6/36 and that they can express this as a percent to one place accuracy (as 16.7%).
- ✦ Have students throw the 6-sided dice 20 times, recording the results. Have students determine the experimental probability (as a fraction and as a percent) of the two numbers showing being the same (a 1 and a 1; a 2 and a 2; etc.). Have them compare the theoretical probability to the experimental probability.

Activity 10: Revisits SET SCENE & addresses achievement indicators 1, 4, 5, 6, and 7.

- ✦ Discuss the ESP task. Organize students into groups. Ask them to create an ESP experiment that involves two independent events. Ask them to determine the theoretical probabilities of "ESPing" correctly.
- ✦ Have the groups share their ESP experiments and conduct them. Discuss whether the experimental probabilities indicate whether a student might or might not have ESP capability.

Activity 11: Assessment of teaching.

- Have students determine the theoretical probability of getting a HEAD and having a '6' showing when a coin and a normal 6-sided die are tossed by listing in a table all possible ways the coin and die can land AND by making a tree diagram. Have them explain why the two events are independent events. [They should determine that the theoretical probability is 1/12.]

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheets.

Question 1.

What is the theoretical probability of picking a black ace from a standard deck of 52 cards? ____________________

Question 2.

Describe an example of two independent events that does not involve dice or spinners. ____

Question 3.

A coin is tossed and a 4-sided die (has 1 to 4 on it) is thrown.

List the sample space.

Draw a tree diagram of the sample space.

Question 4.

A 4-sided die (1 to 4 on it) and a 6-sided die (1 to 6 on it) are tossed. List the sample space . What is the theoretical probability of a sum of 6 showing?

Question 5.

Spinner A, having 1 to 3 on it, and spinner B, having 1 to 5 on it, are spun 20 times in all. A sum of 5 (for the two numbers that the spinners stopped at) occurred 3 times.

a) What is the experimental probability of getting a sum of 5? __________

b) What is the theoretical probability of getting a sum of 5? ____________

Question 6.

Hat A contains the letters A, B, C, once each. Hat B contains the letters A, B, once each, and the letters C and D, twice each. A letter is picked out of each hat and returned. This is done a total of 10 times. The table shows the results.

PICKED LETTERS	NUMBER OF TIMES
B & C	1
B & D	2
B & B	4
A & C	3

a) What is the experimental probability of picking an A & C, expressed as a percent? ____

b) What is the theoretical probability of picking an A & C, expressed as a percent? ____

c) Which probability is greater? ____________

MAINTAIN stage

Mini-task example

Every so often:

- Present probability situation involving two independent events. Ask students to determine the theoretical probability for one of the outcomes, expressing it as a fraction and as a percent.

Rich-task example #1

Have students solve the following problem by listing in a table all possible events (e.g., 10 + 30, 10 + 31; etc.).

A carnival show has a game that involves turning over a card numbered from 10 to 13 inclusive from one row and turning over a card numbered from 30 to 34 inclusive from a second row. The cards are randomly arranged in each row. You win a prize if the sum of the two numbers on the turned-over cards is a prime number. What is the probability of winning a prize?

Rich-task example #2

Have students investigate a monkey typing two-letter words on a restricted typewriter. This typewriter only has the letters from A to H inclusive on it. Have students use a spreadsheet to generate fifty pairs of random numbers from 1 to 8 where 1 corresponds to the letter A, 2 to B, . . . 8 to H. Have students count the number of legitimate two-letter English words generated by the spreadsheet. Have them determine the experimental probability of a monkey typing a legitimate word (number of legitimate words/50) Have students determine the theoretical probability of a monkey typing a two-letter word by listing in a table all possible two-letter entries that the monkey can type and then identifying those entries that are legitimate two-letter words. Have students compare the theoretical probability to the experimental probability.

Comments

These are rich-tasks because each task is a complex problem that integrates probability with other matters.

- ❑ In EXCEL, you generate a random number between 1 and 8 by entering the cell formula, TRUNC(8*RAND()) + 1